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| linear IPS | Year 12 Methods  TEST 3 7 June 2019    TIME: 45 minutes working  **Calculator Assumed**  44 Marks 6 Questions |

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Teacher: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

**Question 1 (5 marks)**

1. Differentiate (2 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 uses product rule  🗸 obtains derivative |

1. Hence find **using** the result in(a) above. (3 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 integrates equation in (a)  🗸 uses fundamental theorem  🗸 uses limits correctly to obtain exact result |

**Question 2 (3 marks)**

Determine the *x*-coordinates of all points on the graph of for where the tangent line is horizontal. (Justify your answers)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 differentiates(must be stated)  🗸 equates derivative to zero  🗸 solves for exact x coordinates within required domain |

**Question 3 (7 marks)**

A survey conducted by a local bank shows that 75% of its customers use an ATM at least once a month.

1. Find the probability that in a random sample of 8 customers, **at least 75%** of them use an ATM machine at least once a month. (2 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 uses binomial parameters and at least 6 successes out of 8  🗸 states probability |

1. If the random variable X follows a binomial distribution with n=12 and p=0.75, what is the mean of this distribution and what is PXmean? (3 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 calculates mean  🗸 uses binomial parameters  🗸 states probability |

1. **If the sample size became very large what would you expect P(** **mean) to approach? Briefly explain your answer. (2 marks)**

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| **Solution** |
| As sample size becomes larger, the distribution becomes more symmetrical about the mean, approaching a probability of 0.5. |
| **Specific behaviours** |
| 🗸 states approaching 0.5  🗸 describes the ideal shape of distribution as sample size becomes very large |

**Question 4 (10 marks)**

The discrete random variable X can only take the values 2, 3 or 4. For these values the cumulative distribution function is defined by



for , where is a positive constant integer.

1. Find the value for (3 marks)

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| **Solution** |
| K equals 1 as k is positive. |
| **Specific behaviours** |
| 🗸 uses  🗸 sets up equation for k  🗸 solves for k and states only a positive value. |

1. Complete the following table for X. (3 marks)

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| **Solution** |
| |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  | 1 | |  |  |  |  | |
| **Specific behaviours** |
| 🗸  🗸 sum of second row equals one  🗸 all entries correct |

1. Hence find and . (2 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 states mean  🗸 states standard deviation |

1. Calculate giving your answer to two decimal places. (2 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 multiplies old variance by positive 4  🗸 rounds to 2 decimal places (only pay this if working is shown for new variance) |

**Question 5 (8 marks)**

Consider the function  where  is in radians.

1. Sketch  on the axes below for  on the axes below.

Clearly label undefined points (if any). (3 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 shape  🗸 open hole at origin or stated undefined at origin  🗸 accuracy with intercepts (within 0.1) |

1. As  approaches zero from the positive side, state the value that approaches.

(1 mark)

|  |
| --- |
| **Solution** |
| Approaches zero |
| **Specific behaviours** |
| 🗸 states approaching zero |

1. As  approaches zero from the negative side, state the value that approaches.

(1 mark)

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| **Solution** |
| Approaches zero |
| **Specific behaviours** |
| 🗸 states approaching zero |

1. Use the above to define a value for as approaches zero, that is the following limit  . (1 mark)

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| **Solution** |
| equals zero |
| **Specific behaviours** |
| 🗸 states equals zero |

It can be shown that  .

1. Using the fact that and the above results, show that .

(2 marks)

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| **Solution** |
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| **Specific behaviours** |
| 🗸 uses values of both limits  🗸 shows that derivative simplifies to required result |

**Question 6 (11 marks)**

A game is played by throwing two standard six-sided dice into the air once. The sum of the uppermost numbers are added together and if the sum is greater than 8 the player wins $5.

Determine:

1. the probability of winning $5 in one game. (2 marks)

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| **Solution** |
| |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | **1** | **2** | **3** | **4** | **5** | **6** | | **1** | 2 | 3 | 4 | 5 | 6 | 7 | | **2** | 3 | 4 | 5 | 6 | 7 | 8 | | **3** | 4 | 5 | 6 | 7 | 8 | 9 | | **4** | 5 | 6 | 7 | 8 | 9 | 10 | | **5** | 5 | 7 | 8 | 9 | 10 | 11 | | **6** | 7 | 8 | 9 | 10 | 11 | 12 | |
| **Specific behaviours** |
| 🗸 recognises that there are 36 outcomes  🗸 states prob (no need to simplify) |

1. the probability of winning exactly $15 in 5 games. (3 marks)

|  |
| --- |
| **Solution** |
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| **Specific behaviours** |
| 🗸 states Binomial  🗸 uses parameters  🗸 states prob |

1. the probability of winning at least $15 in at most 5 games. (3 marks)

(assume that 

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| --- |
| **Solution** |
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| **Specific behaviours** |
| 🗸 examines 3 games with correct parmeters binomialCDf  🗸 examines 4 and 5 games and cumulative values  🗸 states final prob |

1. the minimum number of games to be played so that the probability of winning at least $15 is greater than 0.47. (Justify) (3 marks)

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| **Solution** |
| Min number of games is 9 |
| **Specific behaviours** |
| 🗸 uses cumulative Binomial with correct parameters  🗸 shows at least 3 sets of trials  🗸 demonstrates that 9 games is the minimum |